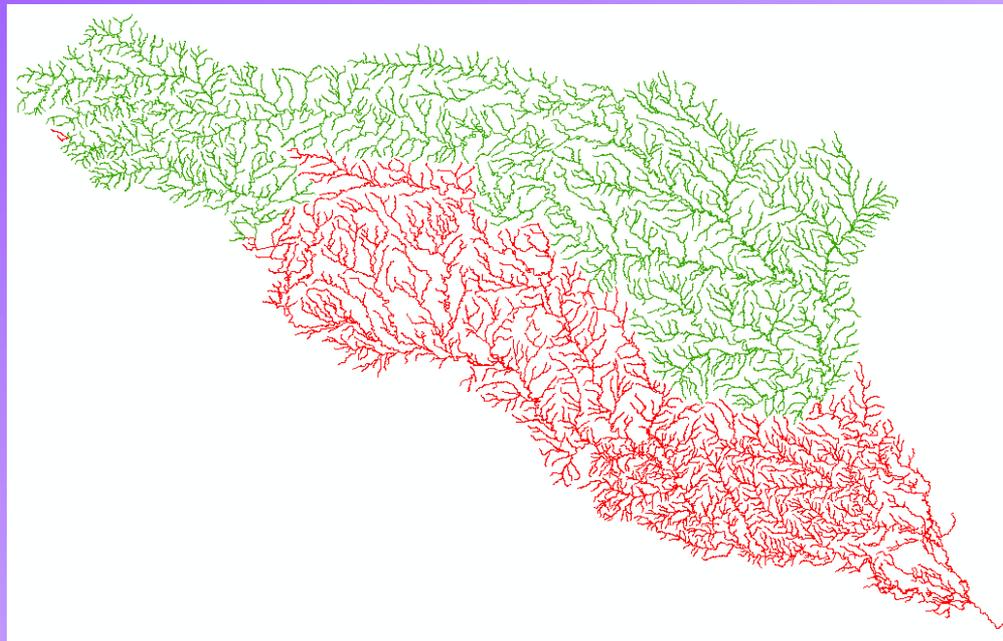


# Parallel Computing in RAPID



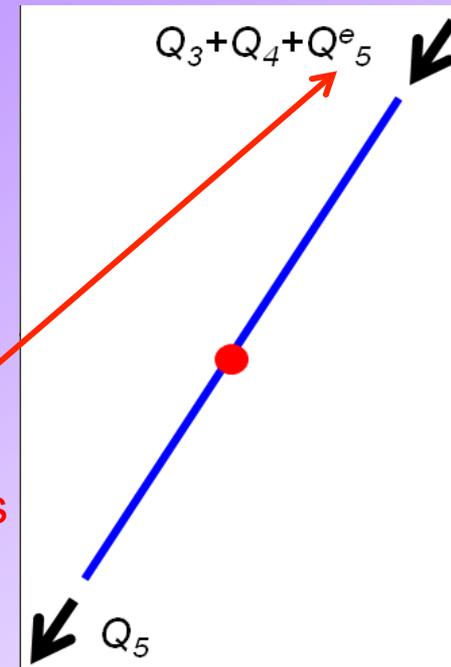
By Cédric H. David  
([cedric.david@jpl.nasa.gov](mailto:cedric.david@jpl.nasa.gov))

18 Jun 2012, updated 21  
Jan 2015

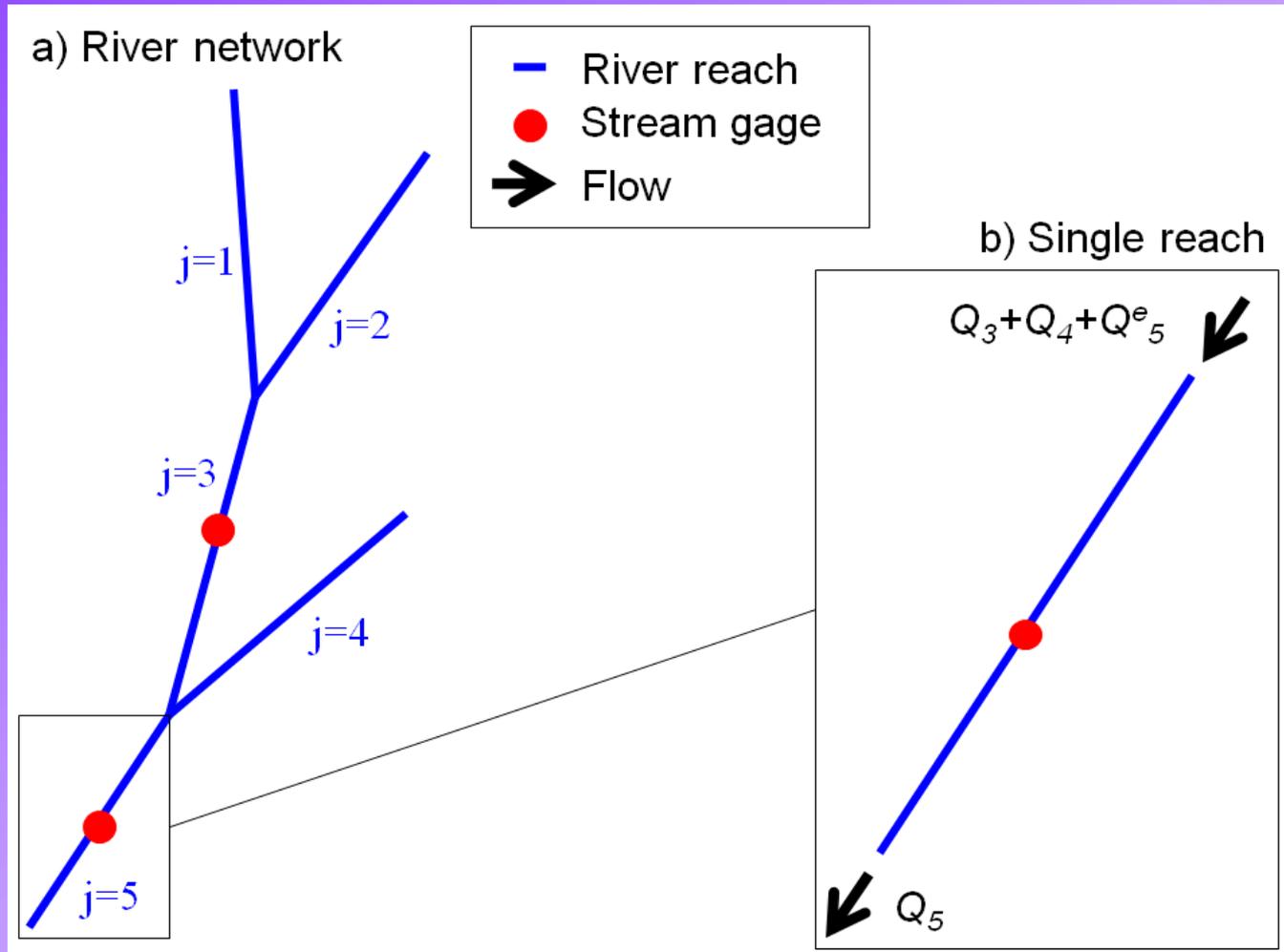
# Muskingum method applied to one river reach

$$\begin{aligned}
 \text{Outflow}(t+\Delta t) & \\
 Q_5(t+\Delta t) &= C_{1_5} \cdot \left[ \underbrace{Q_3(t+\Delta t) + Q_4(t+\Delta t) + Q_5^e(t)}_{\text{Inflow}(t+\Delta t)} \right] \\
 &+ C_{2_5} \cdot \left[ \underbrace{Q_3(t) + Q_4(t) + Q_5^e(t)}_{\text{Inflow}(t)} \right] \\
 &+ C_{3_5} \cdot \underbrace{Q_5(t)}_{\text{Outflow}(t)}
 \end{aligned}$$

Assumptions



# Conceptual river network



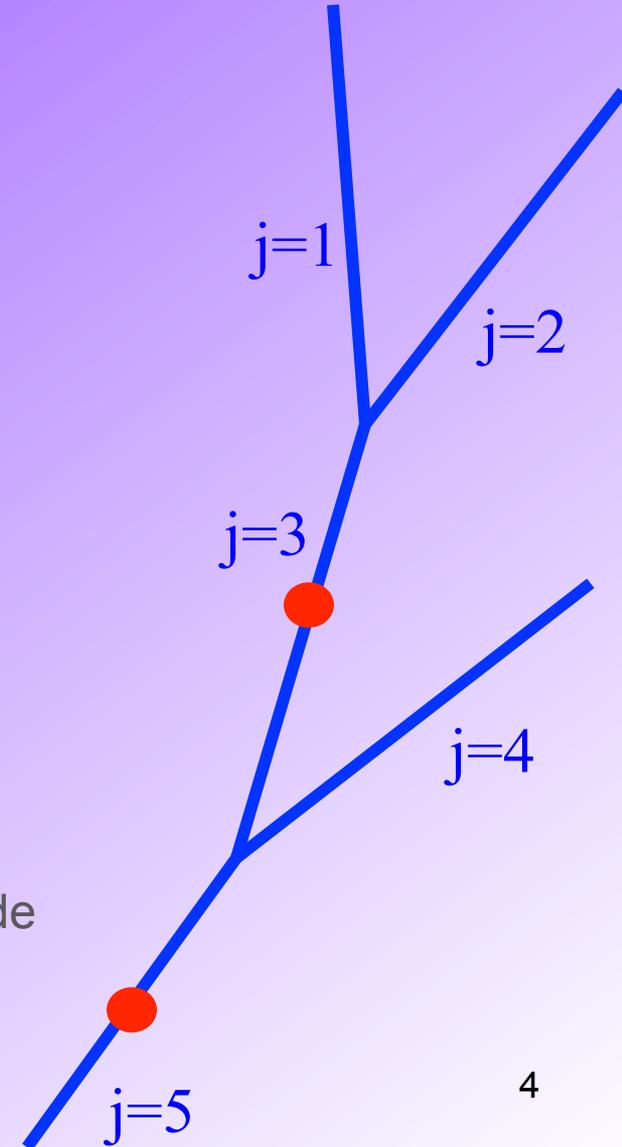
# Muskingum method applied to a river network in a matrix-based approach

$$\begin{aligned}
 (\mathbf{I} - \mathbf{C}_1 \cdot \mathbf{N}) \cdot \overbrace{\mathbf{Q}(t + \Delta t)}^{\text{Flows}} &= \mathbf{C}_1 \cdot \overbrace{\mathbf{Q}^e(t)}^{\text{Lateral inflows}} \\
 &+ \mathbf{C}_2 \cdot (\mathbf{N} \cdot \mathbf{Q}(t) + \mathbf{Q}^e(t)) \\
 &+ \mathbf{C}_3 \cdot \mathbf{Q}(t) \\
 &= \mathbf{b}(t)
 \end{aligned}$$

Parameter matrices

Network matrix

Right-hand side



**Simultaneous computation on entire river network**

# Network and parameter matrices

$$\mathbf{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix}$$

Vector of outflows

$$\mathbf{N} = \begin{bmatrix} & & & & \\ & & & & \\ 1 & & & & \\ & & & & \\ & & & & \\ & & & 1 & 1 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

Network matrix  
 $N_{i,j}=1$  if reach  $j$  flows into reach  $i$

$$\mathbf{C}_1 = \begin{bmatrix} C_{1_1} & & & & \\ & C_{1_2} & & & \\ & & C_{1_3} & & \\ & & & C_{1_4} & \\ & & & & C_{1_5} \end{bmatrix}$$

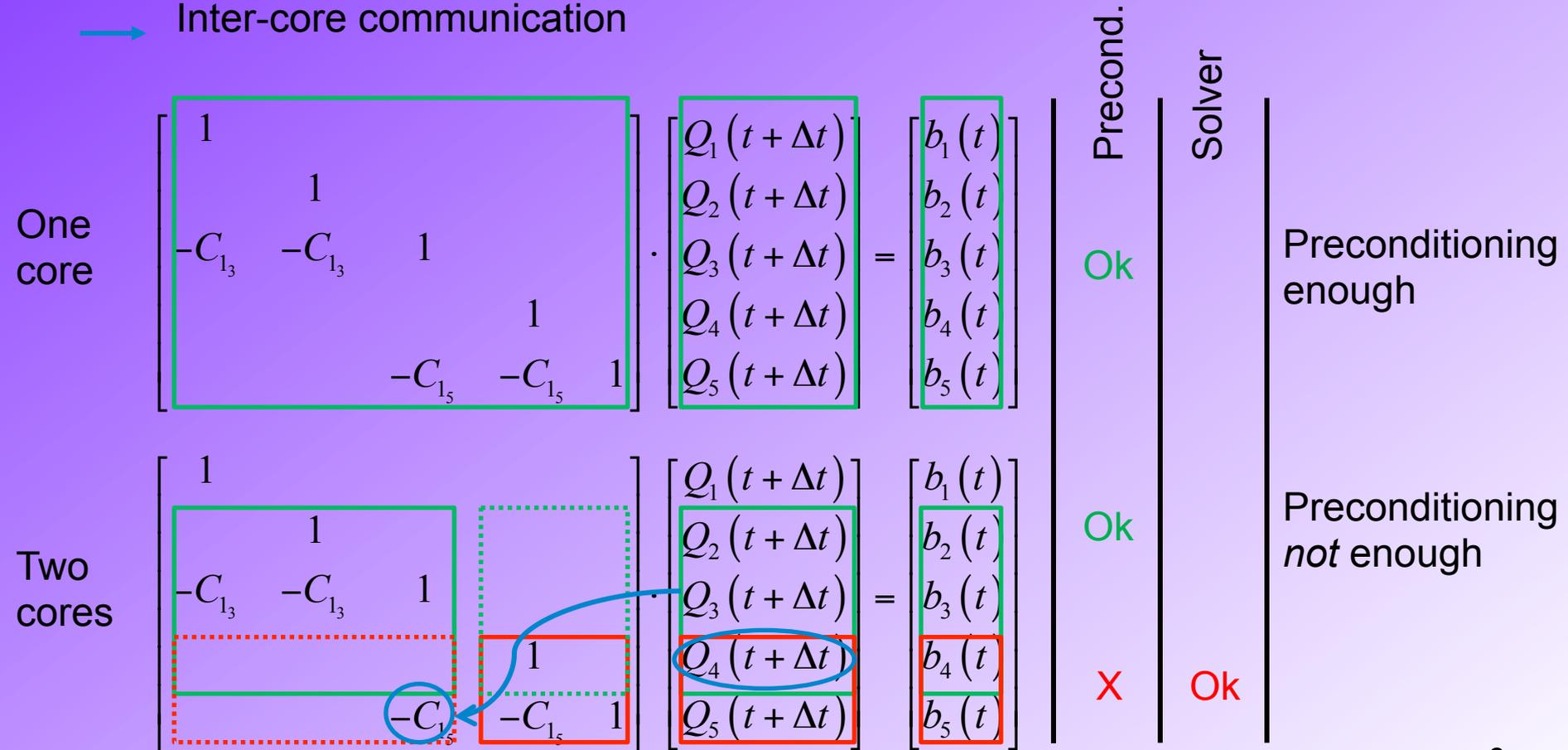
Parameter matrix

$$\mathbf{I} - \mathbf{C}_1 \cdot \mathbf{N} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ -C_{1_3} & -C_{1_3} & 1 & & \\ & & & 1 & \\ & & & -C_{1_5} & -C_{1_5} & 1 \end{bmatrix}$$

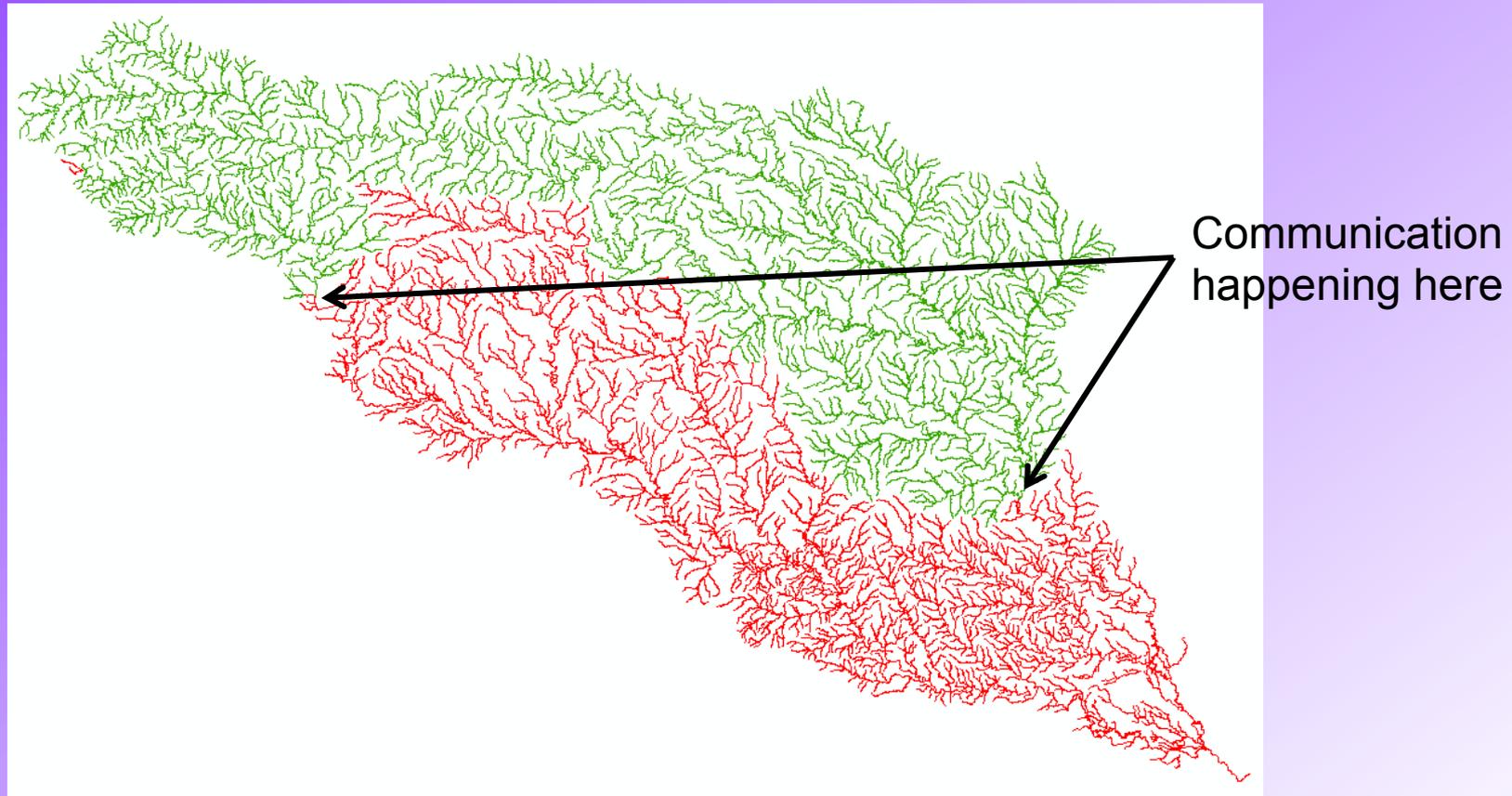
Linear system matrix

# How to solve this linear system on multiple cores?

- Core 1, diagonal block
- Core 2, diagonal block
- Inter-core communication
- ⋯ Core 1, off-diagonal block
- ⋯ Core 2, off-diagonal block



# Example for San Antonio and Guadalupe Basins in Texas



When river reaches are ordered with decreasing Hydroseq

## Further information

RAPID website: <http://rapid-hub.org/>

RAPID source code: <https://github.com/c-h-david/rapid/>